CS166: Final Project

Modeling exploding star system

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Introduction

In this paper, I want to simulate the dynamics of a group of stars (i.e. a star cluster) to see how they interact and merge with each other. The specific situation I am interested in is an exploding group of stars, where all the stars are moving away from their center of mass due to an explosion. I want to estimate the initial velocity needed for the star to avoid collapsing again gravitationally. The second scenario that I am considering is a binary star system, where two same mass stars orbit around each other. In this context, I am interested to estimate the critical transitional velocity they needed to avoid both gravitational collapses or escaping from the orbit.

Assumptions and constraints

For the first scenario, an exploding star system, I assume that initially the stars are normally distributed around their center of mass and they all are moving radially outward from the center of mass with a similar velocity. All the stars are of the same size and have a mass of 1 mass of sun. The specific system I chose has a cluster size of 25 AU and a density of 0.01, which results in around 20 stars using the formula described in the later sections. While in the second scenario of a binary star, I assume that initially the stars only have tangential velocity and no radial velocity. Their initial tangential velocities are always in the opposite direction and it depends on the mass ratio between the stars. The specific system that I chose has a 20 AU distance between the stars and both of them are one solar mass star.

Variables and their update rules

In each step, all the stars' positions and velocities will be updated simultaneously. In order to avoid any energy loss in the orbital motion, I used leapfrog equations to update the position, velocity, and acceleration:

$$a_i = f(x_i) = \sum_{star} Gm_{star}/d_i^2,$$

where d_i is the distance between the two stars

$$x_{i+1} = x_i + v_i \Delta t + 0.5\, a_i \Delta t^2$$

$$v_{i+1} = v_i + 0.5\,(a_i + a_{i+1})\, \Delta t$$

So at initialization each star will measure the distance of all other stars calculate the gravitational acceleration, the acceleration was divided into the x and y component in this way:

$a_x = a \cos \theta = a\, d_x/d$ and $a_x = a \cos \theta = a\, d_y/d$

here d_x and d_y is the x and y component distance between two star and $d = \sqrt{d_x^2 + d_y^2}$

As we already got x_0, v_0, and a_0 in the initialization, in each update, first I update the positions, then update the acceleration using the new position, and lastly update the velocity using the new and previous acceleration using the leapfrog equation. Position and velocity are updated component-wise.

If the distance between two stars becomes smaller than twice one's radius, both stars will merge into a single star. The parameters of the new star become:

$$m_{new} = m_{star1} + m_{star2}$$

$$Radius,\ r_{new} = \sqrt{(4\pi r_1^{\ 2} + 4\pi r_2^{\ 2})/4\pi}$$

$$v_x = (m_1 v_{x,1} + m_2 v_{x,2})/(m_1 + m_2)$$

$$v_y = (m_1 v_{y,1} + m_2 v_{y,2})/(m_1 + m_2)$$

In each update the x and y component of the center of mass updates as follows:

$$com_x = \sum_{i=0}^{n_{star}} m_i x_i \,/\, m_{total}$$

$$com_y = \sum_{i=0}^{n_{star}} m_i y_i \,/\, m_{total}$$

Empirical Model using Simulation

In the simulation I used the following units for the model parameters:

Distance: AU $= 1.5 \times 10^{11}\ m$

Mass: $M_{sun} = 2 \times 10^{30}\ kg$

Velocity: $v_{star} = 3 \times 10^{3}\ m/s$

Acceleration: m/s

Time: second

Gravitational constant, $G = 6.67 \times 10^{-11}\ m^3/(kg \cdot s^2)$

Every cluster initiates with a given size. For the exploding star system, I wanted 95% of data to stay within a radius equal to the cluster size, so I used size/2 as my standard deviation in the normal distribution. The area of the system is then simply $A = \pi r^2 = \pi(size/2)^2$. The number of stars in the system is calculated from the given density parameter:

$n_{star} = round(density \times A)$.The velocities are also chosen from a narrower normal distribution with a given mean. We used a distribution instead of a single number because in reality even if the stars were thrown away from the center with equal velocity, there would be some randomness and velocity loss and it is not possible that every star would have exactly the same velocity.

Similarly, for the binary stars, the velocity is drawn from a narrow normal distribution binary with the same mass. The given parameter mass ratio determines the ratio of the velocity of the stars. Star 1 is by default the massive star, in the simulation a mass ratio of m is defined as: $m = mass_1/mass_2 = m_1/m_2$. So m is always equal to or higher than 1. The parameter v_mean is the mean of velocity distribution for the less massive star, i.e. star 2: $v_2 = N(\mu_v, \sigma_v)$. The velocity of a star in a binary system depends on the mass of the other star:

$$v_1 = \sqrt{Gm_2/d} \text{ and } v_2 = \sqrt{Gm_1/d}$$

$$\Rightarrow v_1/v_2 = \sqrt{m_2/m_1}$$

$$\Rightarrow v_1 = v_2\sqrt{1/m} = N(\mu_v, \sigma_v)\sqrt{1/m}$$

The radii are calculated using, $radius,\ r_i = (m_i \times 4\pi r_{initial}^2)/4\pi$, where having a mass of m is similar to merging m solar-mass stars.

To ensure a stable change in position in each timestep, timesteps are chosen based on the current acceleration and velocity. The position update rule is $\Delta x = v\Delta t + 0.5\, a\, \Delta t^2$. So following this equation I chose a timestep of, $\Delta t = min(\Delta x/v_{max}, 2\Delta x/a_{max})$, where$\Delta x$will be the most likely change in position. I choseΔx= 0.1 AU in the simulation using the trial and error method. I tried to run the system with different configurations to check if the simulation is working correctly. The simulation was able to get the stable orbits of a binary system with a similar mass and another with a different mass.

Critical Velocity Estimation

In order to estimate the critical velocity needed to avoid gravitational collapse in the exploding star system, I used two metrics. First, I measure the mean and median of the distances of the stars from their center of mass after running simulation until the time of the system reaches 10^8 seconds. I measured these values for different initial velocity and for each velocity I run 20 simulations with a cluster size of 25 AU.

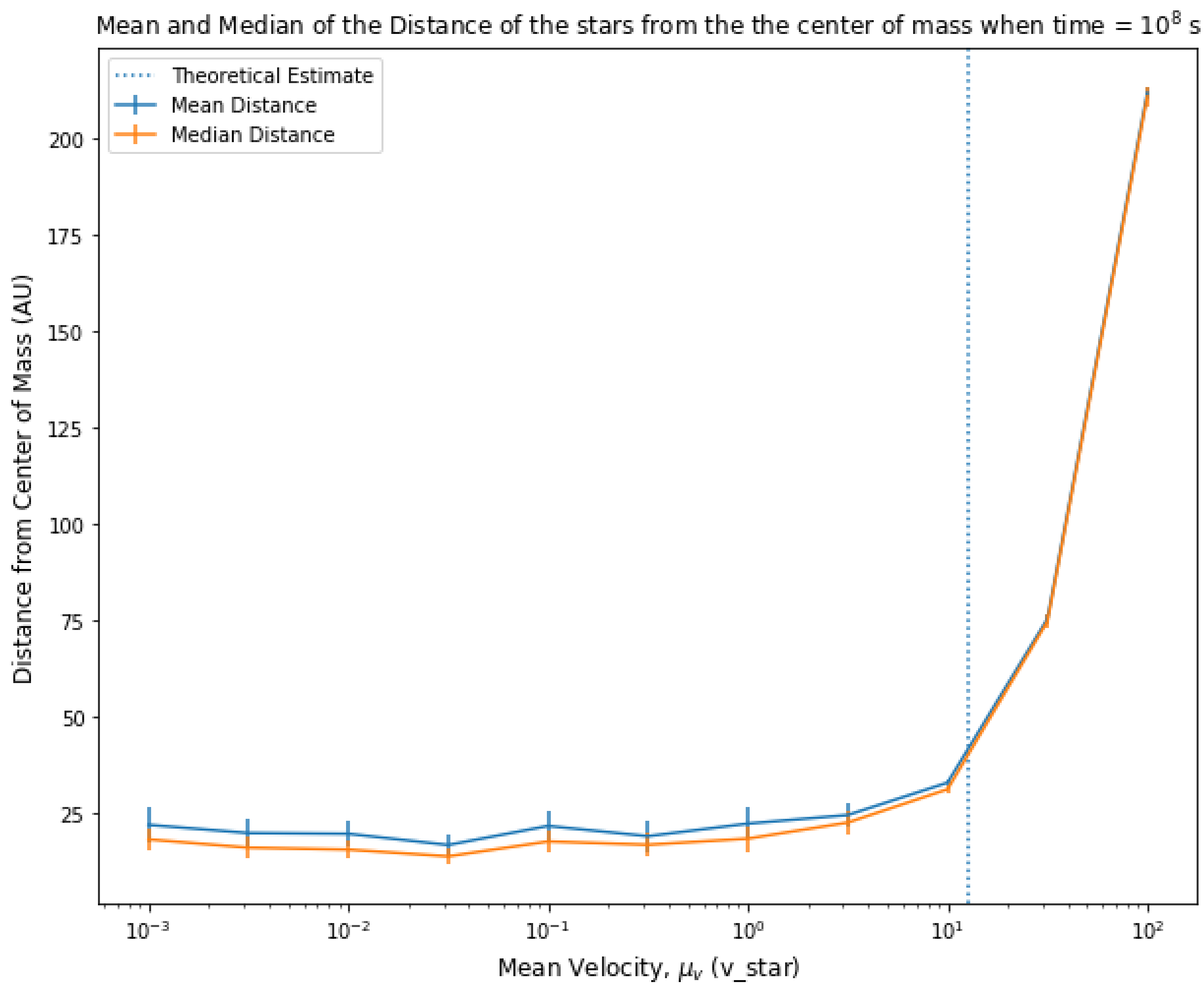

Figure 1: Mean and median distance of the stars from the center of mass shows a stable value around 20-25 AU for low velocities, while in high velocities, the distance rises sharply to 200 AU and with a lower uncertainty suggesting a divergence of stars.

Fig 1 shows that for a smaller initial velocity, the mean and median distances stay close to 25 AU, which suggests that the stars are not going away from each other, but staying within the cluster radius. To interpret, we can say that one possibility is that one or few stars are orbiting the merged massive stars in the center and thus maintaining a stable mean distance. Another explanation can be that if we wait more time, then eventually they all will merge into one single star for many of the simulations. But what is interesting is that once the velocity increases, from some point around 10 v_star, the mean and median distance start to rise very sharply. This suggests that the stars are going away from each other and the more velocity we have the steeper the slope becomes. Also, the confidence interval is narrower for these high velocities, i.e. the mean radius is the same for almost all the simulations and they do not fluctuate. This shows that stars are not interacting with each other much, instead, the value is determined mainly by the initial velocity and the cutoff time(10^8s). So we can say that for these high velocities, most likely the stars will not come back again in a single cluster. And from the graph, we can estimate the cutoff velocity as 10 v_star, from where the sharp rise started.

The second metric that I used is the average fraction of stars that merged after a specific time ($10^{12}s$). I again run the simulation 20 times for each velocity and for varying velocities. Fig 2 shows that for smaller velocities, almost 80% of the stars have been already merged together. Then again once the velocity increases, there is a sharp decline in the fraction of merging to even less than 20%, suggesting that stars are going away from each other and do not collide anymore. So for any velocity equal to or less than $10^{0.5}$v_star, the system will surely converge and for the velocities higher than $10^{1.5}$the system will go away. For a velocity of 10v_star, we can see that 50% of the stars merged together, but still, the mean radius stays close to 25 AU.This can either mean that the system will converge as stars are still staying close by, but it just takes a long time as it is in the critical point.

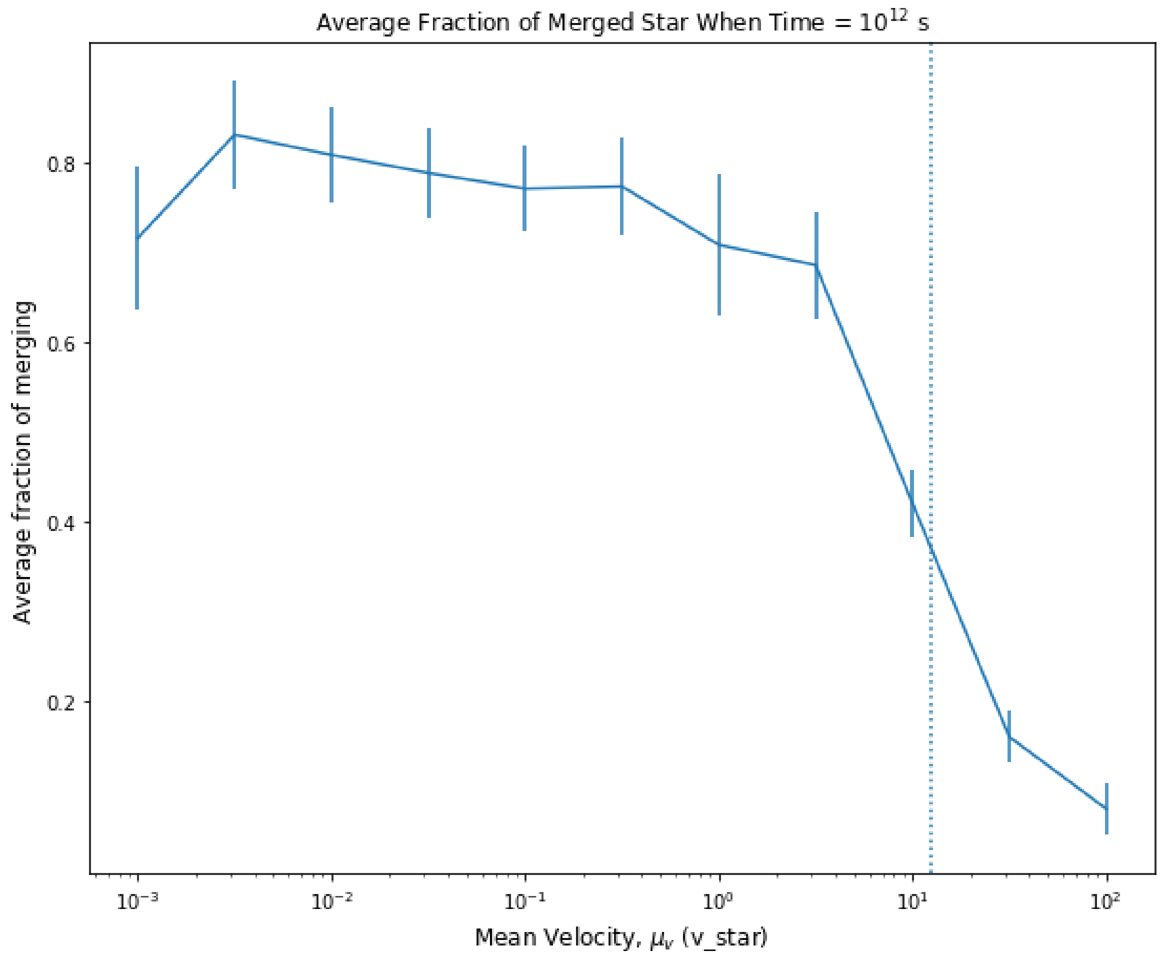

Figure 2: Average fraction of merged star shows a very high value for lower initial velocity as they merge together and converge, while at higher velocities it diverges, thus the fraction is much smaller.

It can also be that the system is slowly diverging away and along the way, some of the stars merged coincidently. But the second explanation is less likely because if it is just coincidental merging, then the confidence interval should be wider, but we can see a narrower interval compared to the low-velocity systems. So most likely they will still converge at 10 v_star and for any higher velocities, they will go away.

In the binary system, I wanted to estimate the velocity needed for a stable orbit where the stars are 20 AU apart and have 1 solar mass each. The metric I used is the distance between the star after a specific time limit (10^{10}s) or after the distance reached a maximum chosen value (500 AU). If the velocity is too low, then they will merge together and the distance will be 0 between them. If the velocity is too high, then they will continue to go away from each other and the distance will be minimum. In the middle, for some optimal velocity, we can have a stable distance.

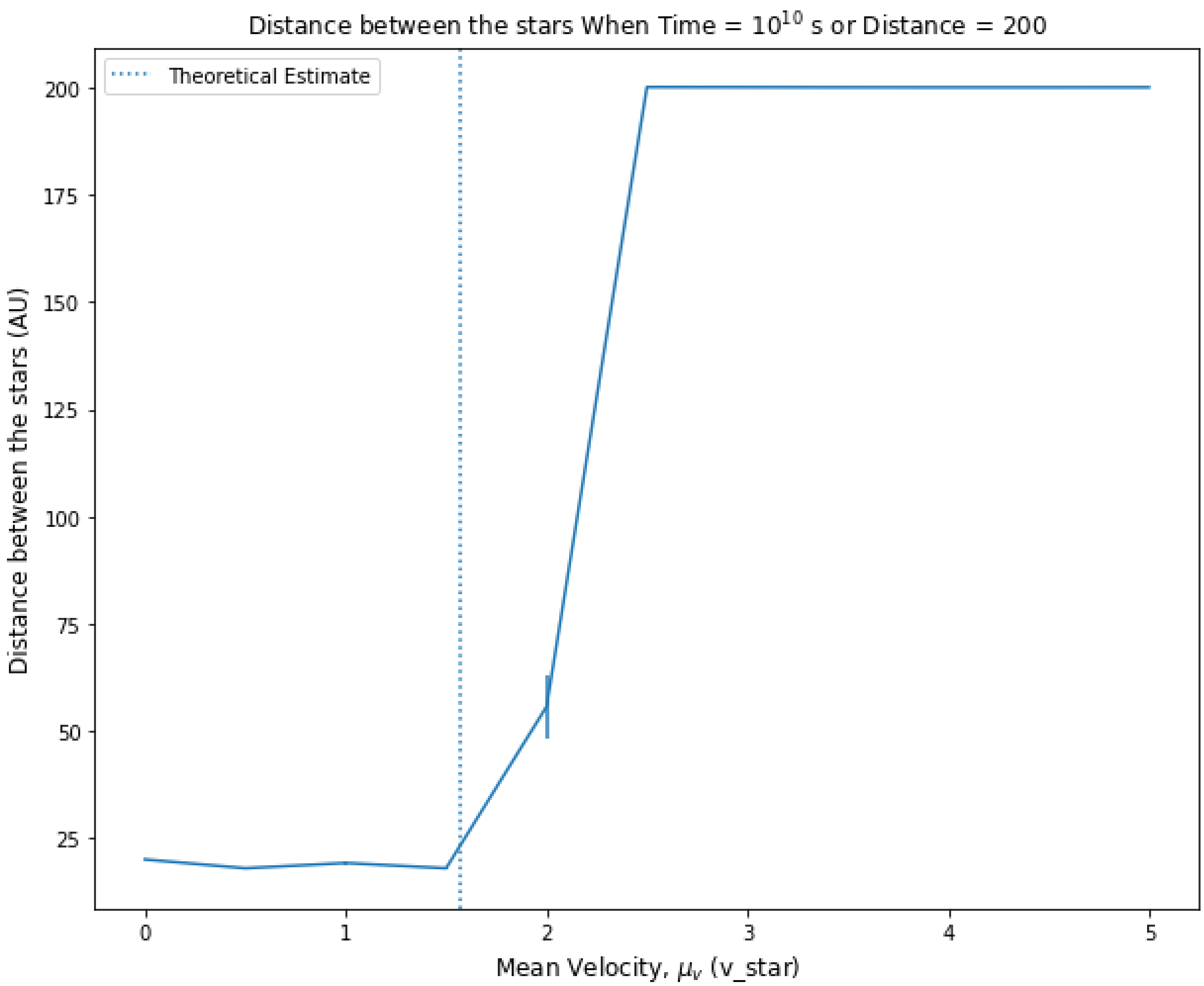

Figure 3: The distance between the stars when time is $10^{10}$for varying initial velocity shows that at high velocity, it go away from each other, thus the distances increases a lot, while at lower velocity it either orbits or still in the process of merging, thus have a low distance.

Fig 3 shows that for 2.5 v_star or higher velocity, they just go away from each other, while for lower than 1.5 v_star velocities, the distance is around 20-25 AU, which should be the distance if they orbit around each other. But one reason why we are getting the same distance even for a very small velocity can be because of the time limit. Maybe the stars are not orbiting, rather they are going to merge, but it will just take longer than 10^{10} seconds. Nevertheless, if it really orbits around each other, their distance should not rise very much than 20-25 AU. So we can say that 1.5 v_star is the limit where we still orbit the star. At 2 v_star, it has a value of 50 AU with a high variance as the confidence interval is higher. It is also possible that maybe they still orbit at 2 v_star, but with a very eccentric orbit. So the estimated cutoff should be between 1.5-2 v_star.

Theoretical Model and Comparison

For the exploding system, we can estimate that the mass of the full system, which is 20 M_sun, is in its center of mass, and the average distance of a single star is equal to the size of the cluster = 25 AU. Now to escape from a system, the kinetic and potential energy must sum to 0:

$$E_{potential} + E_{kinetic} = 0$$

$$-G(M_{com})\, m / d + 0.5\, m\, v_{esc}^{2} = 0$$

$v_{esc} = \sqrt{2GM_{com}/d} = \sqrt{2G(20\, M_{sun})/25\, AU}$/ v_star ~ 12.57 v_star

We can see that this is pretty close to our empirical estimate of 10 v_star. One reason why the empirical value is slightly smaller is that the masses are not distributed only in the center of mass, but they are scattered around the region. So it is possible that the combination of force will not move a particle to the center of mass, rather it will move it in another direction. As a star is not getting similar force towards the center as it will get for a closed system, it can escape even with a lower value.

For Binary Stars, as their mass is equal, the center of mass should be in the middle of the system, d/2 away from each star. In order to orbit around the center of mass, the gravitational force must be equal to the centripetal force needed for rotation:

$$m_1 v^2/(d/2) = G m_1 m_2/d^2$$

$$\Rightarrow v = \sqrt{G m_1/2d} = \sqrt{G(1\ M_{sun})/2(20\ AU)}/v_{star} \sim 1.57 \text{ v\_star}$$

We can see that it is very close to the estimated empirical value. One reason why the theoretical line did not cross at 25 AU is that we assume that the orbit is circular, which it will be if their initial velocity is the same. But in the simulation the velocity are drawn from a normal distribution, thus they can vary a little, which can make elliptical orbits and thus higher distances.

Appendix

Notebook Link:

https://colab.research.google.com/gist/mahmud-nobe/752895050db5cc158d1b4523f20ae1e4/cs166_final_project.ipynb